ECOLE DES MINES D'ALBI
C A R M A U X

RHEOLOGY AND MIXING OF SUSPENSION AND PASTES

Pr Ange NZIHOU, EMAC France

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PLAN

1- Rheology and Reactors

Reactor performance problems caused by rheological behaviors of suspensions et pastes

2- Rheology of complex fluids

Definition
Classification of mixtures
Non-Newtonian behaviors
Behavior laws of viscoplastic fluids
Thixotropy
Viscosity equations
Rheological measurements

3-Factors influencing the rheological behavior of fluids

4- Mixing of pastes in agitated vessels

Agitator and utilization
Geometric parameters
Dimensional numbers
Dimensionless numbers

1- Rheology and Reactor

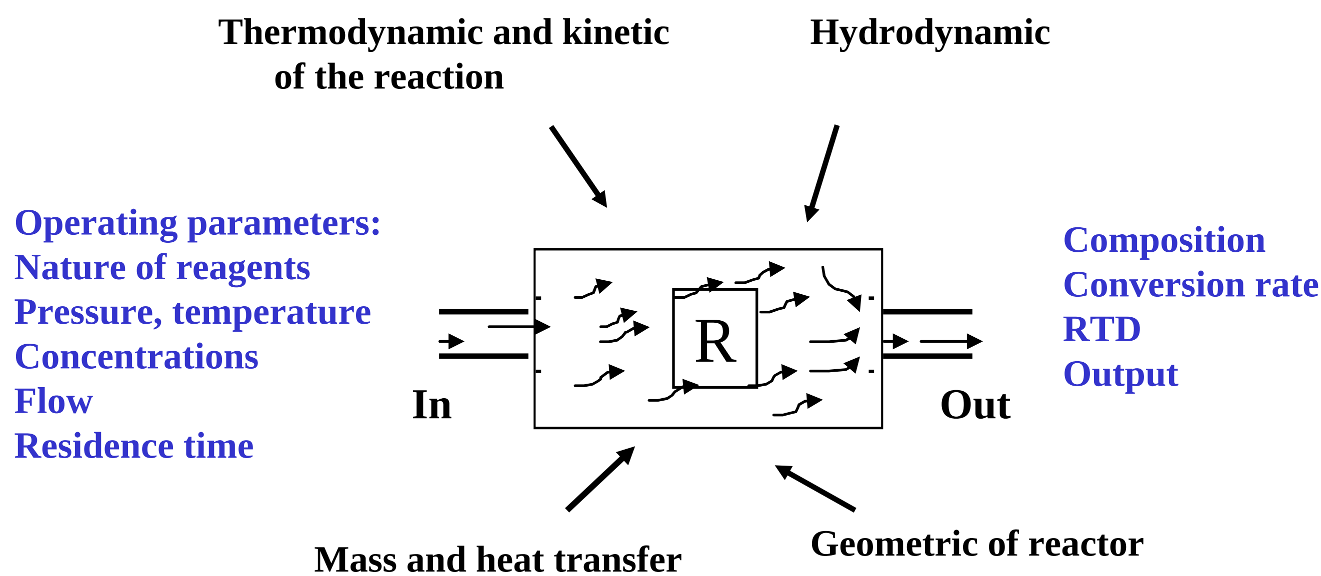
DESIGN OF REACTOR FOR SCALE UP

Mass balance:

$$\begin{pmatrix} A_{j,in} \\ Flux \end{pmatrix} + \begin{pmatrix} A_j \\ \\ \mathrm{Pr}\,oduction \end{pmatrix} = \begin{pmatrix} A_{j,out} \\ Flux \end{pmatrix} + \begin{pmatrix} A_j \\ Accumulation \end{pmatrix}$$

DIMENSIONS OF REACTOR IN VIEW OF SCALE CHANGE

PERFORMANCE OF REACTOR:

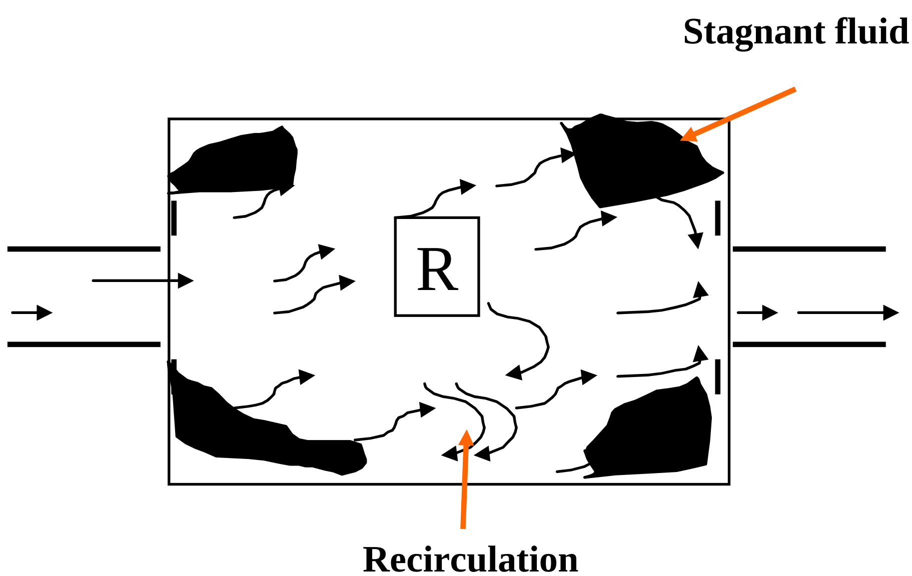


SIMILARITY PRINCIPLE:

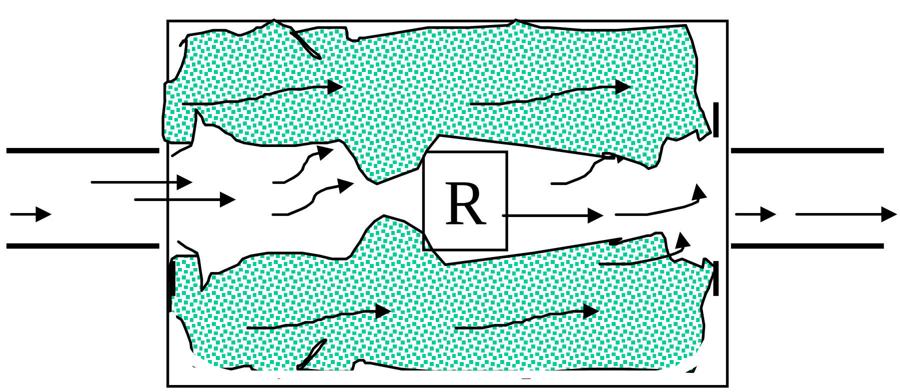
- ❒ Geometric similitude
- ❒ Kinematic similitude
- ❒ Energetic similitude
- ❒ Thermal similitude

ENCOUNTERED PROBLEMS WITH REACTOR

- Existence of dead matter and recirculation:



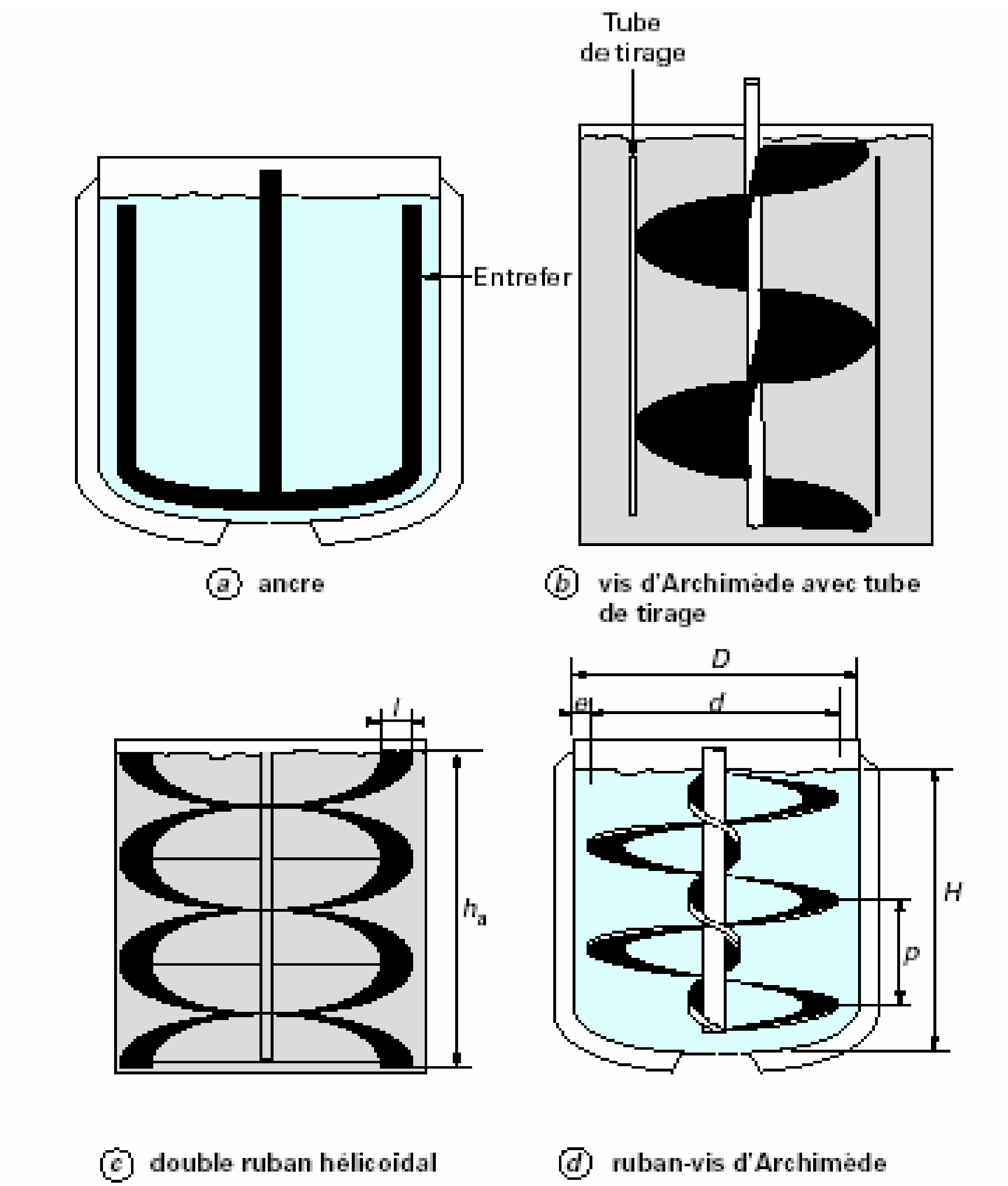
- Presence of preferred passages

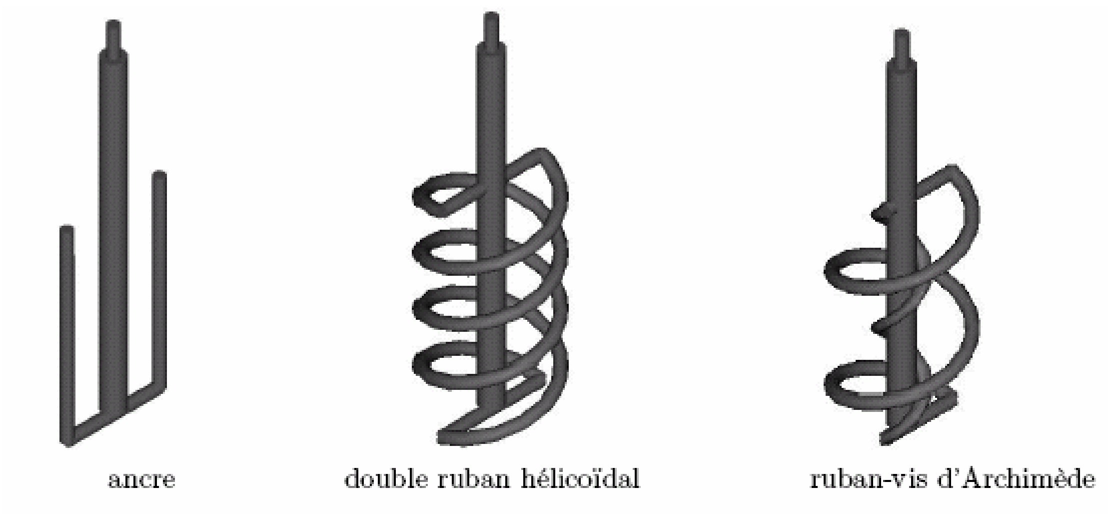


- OBJECTIVE:

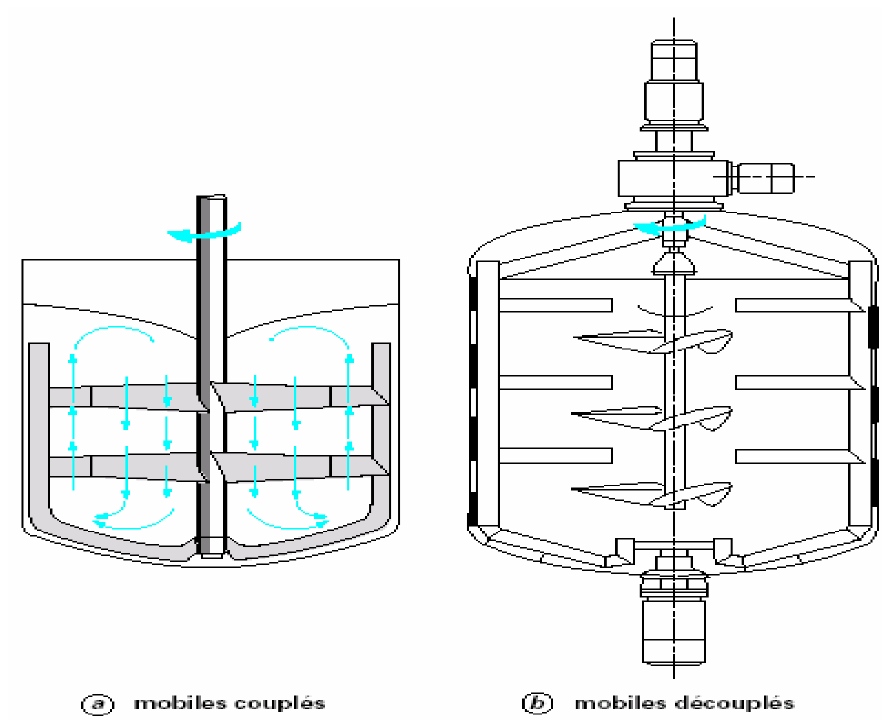
Correct the flows or take it into consideration while designing the reactor

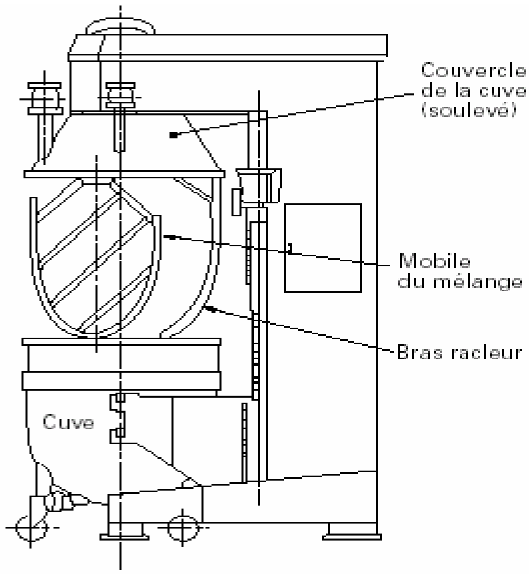
Ribbon impellers (agitators) for mixing Complex fluids



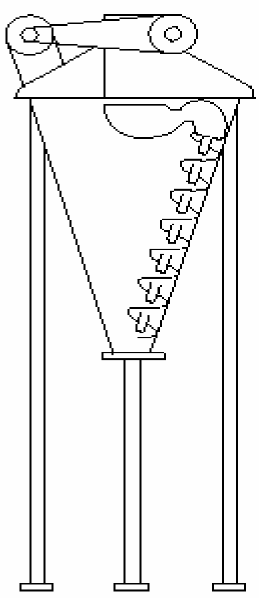


anchor Helicoidal ribbon Archemedian ribbon impeller

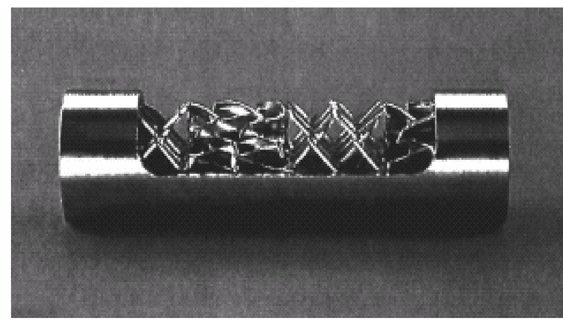




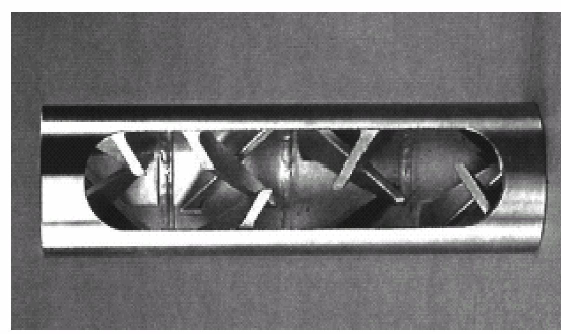
(a) **mobile simple**
(doc. Rayneri)



(b) ***Nautamix*** **[68]**



(a) **Sulzer SMX** (doc. Sulzer-Frères)



(b) **Sulzer SMF** (doc. Sulzer-Frères)

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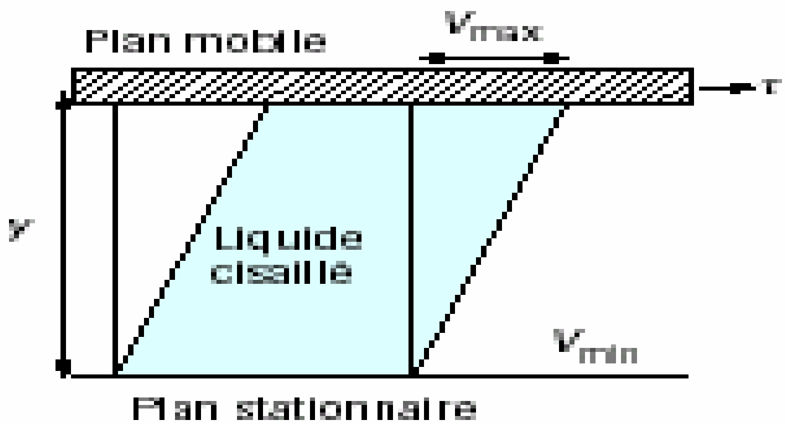
2- Rheology of complex fluids

Model of flowing fluid between 2 plates in which one is mobile (upper plate) and the other is motionless (lower plate)



$$\tau_{xy} = \frac{F}{S} = \eta \frac{\partial V_x}{\partial y} = \eta \dot{\gamma}$$

τ_{xy} Shear stress

$\dot{\gamma}$ Shear rate

η Dynamic or absolute viscosity coeficient

This rheological equation depends on the nature of the fluid and external conditions (T et P)

$$\tau = \eta_a \dot{\gamma}^n$$

n: Behavior index

η_a Apparent viscosity

Characterization of the rheological behavior of fluids using **rheograms**:

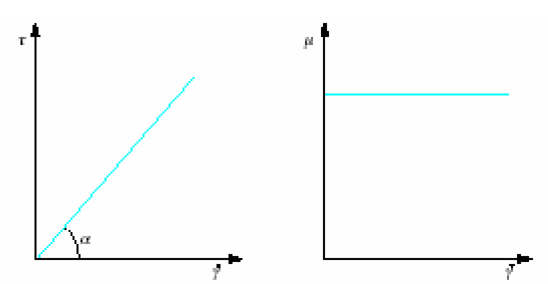
-Graph representing the shear stress vs the shear rate

$$\tau - \dot{\gamma}$$

- Graph representing the shear stress vs the deformation

$$\tau - \gamma$$

1- Newtonian behavior



$$\tau = \eta_a \dot{\gamma}^n \qquad n = 1, \eta_a = \eta$$

Viscosity laws

Several models are available in literature including those for:

1- Homogenous fluids

- Carreau's model

To define the characteristic time of the media

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \left[1 + \left(t_B \dot{\gamma}\right)^2\right]^{\frac{n-1}{2}}$$

with $\eta = \eta_\infty, \dot{\gamma} \to \infty$

$\eta = \eta_0, \dot{\gamma} \to 0$

t_B, Characteristic time

- Ellis' model

$$\frac{\eta_0}{\eta} = 1 + \left(\frac{\tau}{\tau_{1/2}}\right)^{\alpha - 1}$$

$\tau_{1/2}$: Shear stress for $\eta = \frac{\eta_0}{2}$

α : Ellis' parameters that depends to the behavior index

2- Biphasic Fluids

Examples: Suspensions, pastes

When the proportion of the solid is taken into account through the volume fraction ϕ:

$$\phi = \frac{\frac{x}{\rho_s}}{\frac{x}{\rho_s} + \frac{1-x}{\rho_l}}$$

X: the concentration of solid

ρ_s : density of the solid phase

ρ_l : density of the liquid phase

For a dense and random packing of particles in the liquid phase:

$$\phi_{max} = 0,64$$

Behavior Laws:

- For diluted suspensions of spherical particles:

$$\eta = \eta_l (1 + 2,5\phi) \quad \text{Einstein's Law}$$

- For high values of ϕ:

$$\eta = \eta_l \left(1 + \frac{\phi}{\phi_m}\right)^{-q} \quad \text{Krieger-Dougherty's Law}$$

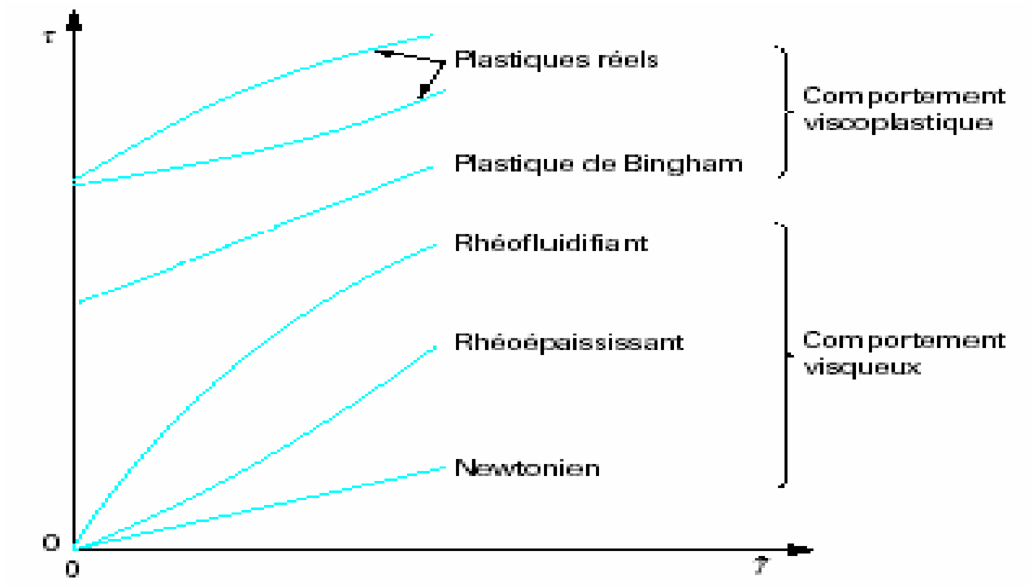
$$\eta = \eta_l \left[1 + 0,75 \frac{\frac{\phi}{\phi_{max}}}{\left(1 - \frac{\phi}{\phi_{max}}\right)}\right]^{B} \quad \text{Loi Chong et al.}$$

With B=2

If B is not a constant:

$$\eta = \eta_l \left[1 + A \frac{\frac{\phi}{\phi_{max}}}{\left(1 - \frac{\phi}{\phi_{max}}\right)}\right]^{B(\tau,\dot{\gamma})} \quad \text{Nzihou et al.}$$

2- Non-Newtonian behaviors



2.1- Viscious behavior

$$\tau = \eta_a \dot{\gamma}^n$$ Ostwald De Waele's Law

n= 1, Newtonian fluid
n< 1, Shear thinning fluid
n> 1, Shear thickening fluid

2.2- Viscoelastic behavior

$$\tau = \tau_0 + \eta_a \dot{\gamma}^n$$ Hershell-Bulkley's Law

τ_0 : Yield stress

n= 1, Bingham's fluid and $\eta_a = \eta_B$

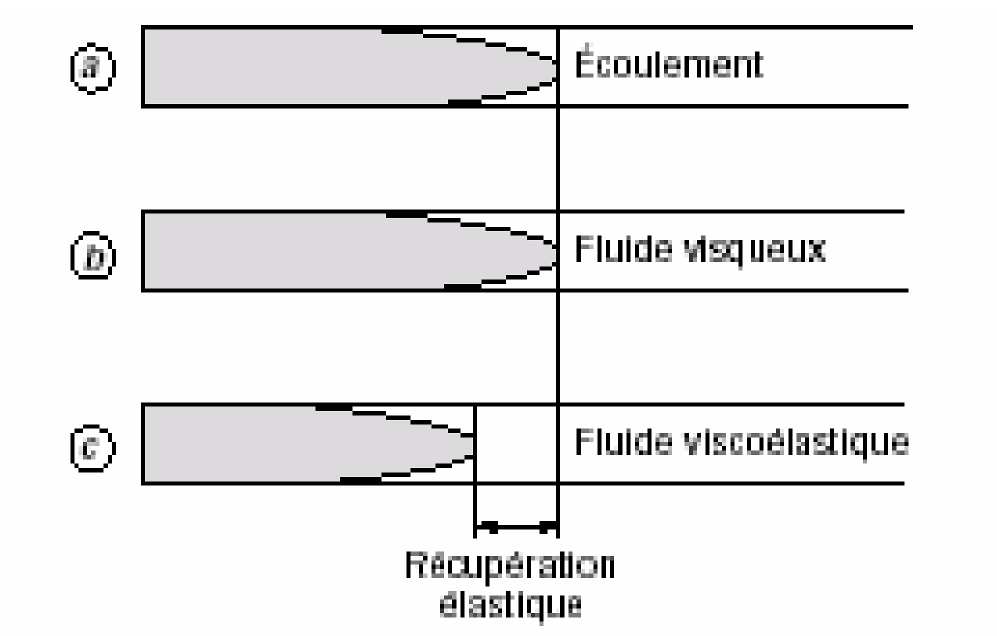
η_B : Bingham's plastic viscosity

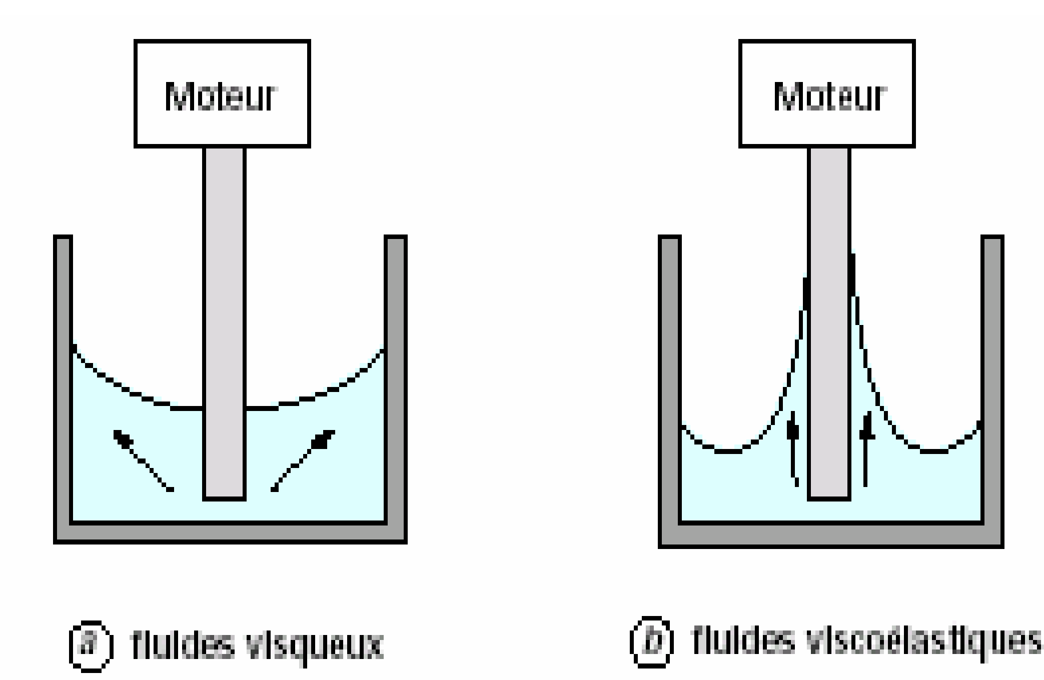
For a number of food and cosmetic fluids:

$$\sqrt{\tau} = \sqrt{\tau_0} + \sqrt{\eta_c \dot{\gamma}}$$ Casson's Law

η_c : Casson's plastic viscosity

Manifestation of different behaviors

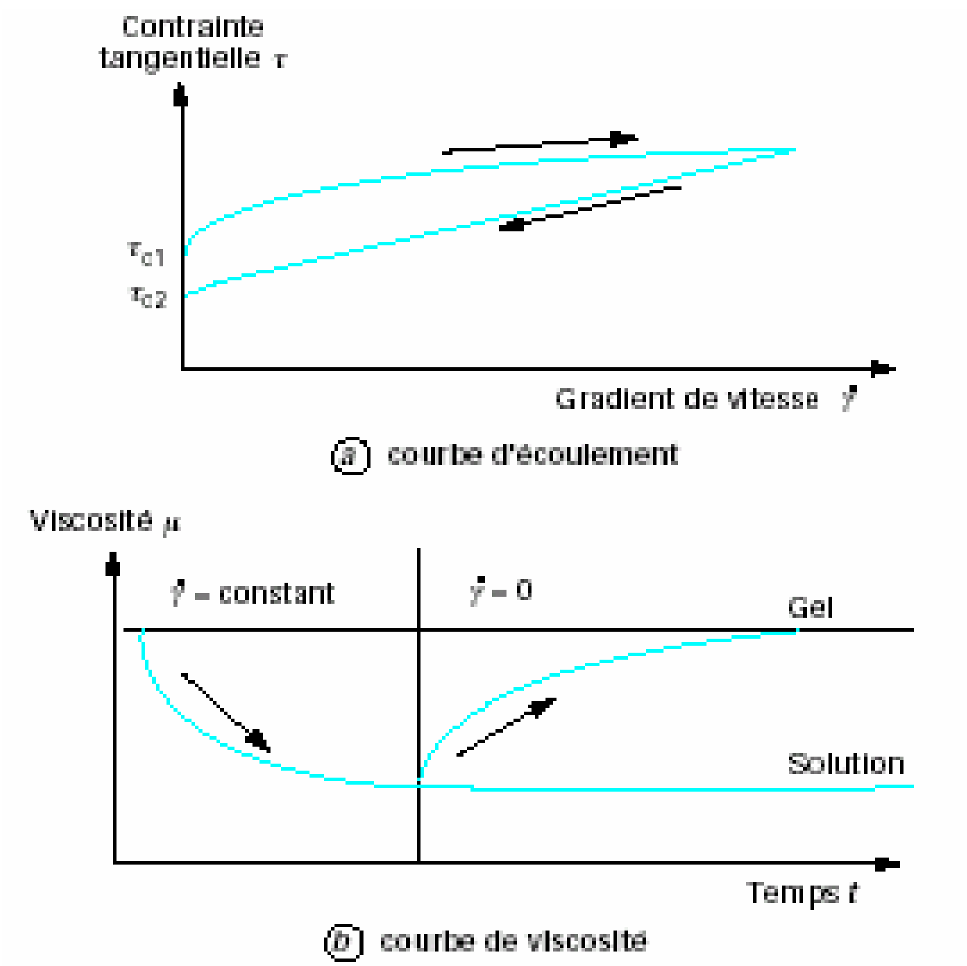




3- Thixotropic Behavior

Time effect in non-newtonian fluid

This is a reversible process.



Origin of the behavior: Breakdown, equilibrium, rebuilding

Representation of the rheological behavior of thixotropic fluids

Behavior of law:

$$\tau = (\tau_0 + \lambda_c \tau_s) + \eta_B \dot{\gamma}^n$$

τ_s : Structure stress

$\lambda_c = 1$ Structure at rest

$\lambda_c = 0$ Complete breakdown of the structure at a high shear rate

Formation rate breakdown of the thixotropic structure:

$$\frac{d\lambda_c}{dt} = a(1 - \lambda_c) - b\lambda_c \dot{\gamma}$$

a and b are specific parameters of the mixture.
They must be determined experimentally.

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3- Important factors influencing the rheological behavior

To be discussed during the course

- Density and volume fraction of solids

- Porosity of the solid

- Particle size distribution

- Form/Shape of particles

- Surface area

- Interfacial properties (chemical composition and structure)

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4.2 Geometric parameters

H/D, n_R, I/d or I/D, d/D, p/d or p/D, h_a/d, e/D, w/d, Y/D, L, n_p

d, diameter of the agitator (m)
D, interior diameter of the reactor (m)
e, gap (m)
H, height of the suspension in the reactor (m)
ha, total height of the the agitator (m)
I, width of ribbon (m)
L, length of blades (m)
n_R, number of ribbon
n_p, number of blades
p, helix gap

Example: Archimedean screw impeller

d/D=0,95
0,5< p/d <2
0,044< I/D <0,33
40< Re_e <270
0,023< e/D <0,097
n_R = 1

4.3 Dimensionless numbers

The Reynolds number in effect in the agitation

Representation of the flow regime (the inertia and viscous effect)

$$Re_e = \frac{\rho N d^2}{\mu_e}$$

N, rotation speed (s^{-1}), d agitator's diameter (m),
μ_e effective viscosity of suspension (Pa.s) obtained with the rheometer, ρ density of the suspension in kg/m^3

Flow regimes :

$Re_e < 10\text{-}50$: laminar flow
$10\text{-}50 < Re_e < 10^4$ intermediate flow
$Re_e > 10^4$ turbulent flow

4.4 Dimensional number

Agitation power

Necessary driving force for the agitator

$$P = K_p \mu_e N^2 d^3$$

Calculation of Kp, constant of helical mobile.
A few equations:

$$k_p = 66 n_R (p/d)^{-0,73} (e/d)^{-0,6} (I/d)^{0,5} (H/d)$$ Hall’s correlation

$$k_p = 52,5 n_R^{0,5} (p/d)^{-0,5} (e/d)^{-0,5}$$ Nagata’s correlation

$$k_p = a_M (p/d)^{0,7} (I/d)^{-0,03} \mathrm{Re}^{b_M}$$ Archimedean screw ribbon

Effective shear rate

$$\dot{\gamma}_e = K_s N$$

Ks, Metzner and Otto's constant given by the following correlations.
A few correlations:

For 0,026< e/D <0,16 Ks=34 –114(e/d) Shamlou's correlation

Ks = 8,9(e/D) $^{-1/3}$ Kuriyama's correlation

$$k_s = 25(d/D)^{0,5}\left[\left[\frac{(p/d)}{\pi^2 + \left(\frac{p^2}{d^2}\right)}\right]^{0,5}\right]^{-0,15}$$ Bakker's correlation

If 0,023< e/D >0,097 ; 0,91> p/D <1,9 ; 0,077<I/D <0,2, we have :

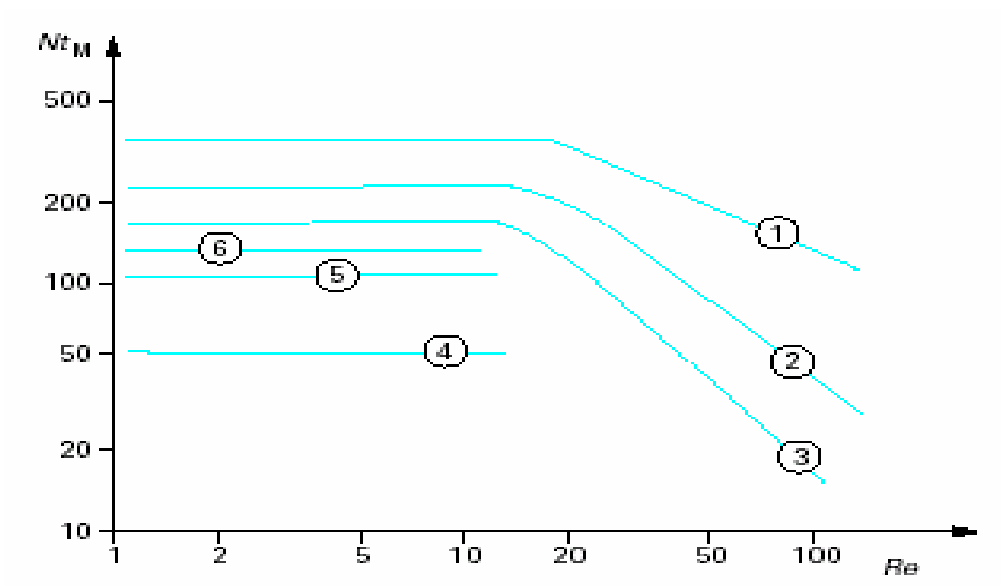
$$k_s = 38,3(0,814)^{1/n}(p/d)^{-0,14}(I/d)^{-0,024}$$ Yap's correlation

with n, the behavior index determined by the rheometer

Mixing time

$$N = \frac{Nt_M}{t_M}$$

To be discussed during the course (see the graph)



(1) ancre (*d* /*D* = 0,98) - fluide newtonien

(2) vis hélicoïdale (*d* /*D* = 0,62) sans tube de tirage - fluide newtonien

(3) vis hélicoïdale (*d* /*D* = 0,62) avec tube de tirage - fluide newtonien

Influence de la rhéologie du fluide avec le même ruban hélicoïdal

(4) ruban hélicoïdal - fluide newtonien

(5) ruban hélicoïdal - fluide rhéofluidifiant

(6) ruban hélicoïdal - fluide viscoélastique

4.5 Heat transfer (Nusselt Number)

It is necessary to clear the flux of reactive heat as well as the heat generated by the agitator which can reach several kW/m^3 for suspensions having an high effective viscosity. The heat-exchange surface coefficient and therefore Nusselt's equation, Prandl's equation, the dissipated heat, the volumic capacity of cooling, the exchange coefficient agitated-wall suspension can be determined.

Heat transfer

Dimensionless numbers: Nusselt's equation, Prandtl's equation, Reynolds' equation, are defined in order to establish the relation between different system variables and the importance of certain phenomenon in relation to others.

- Nu corresponds to the relationship between the transport of heat by conduction-convection and the transport of heat by conduction

$$Nu = \frac{hD}{\lambda}$$

-Prandtl's equation represents the relationship between the molecular diffusivity of the matter and the molecular diffusivity of heat.

$$Pr = \frac{C_p \mu}{\lambda}$$

- Reynolds' equation represents the relationship between inertial forces and viscous forces.

$$Re = \frac{\rho u D}{\mu}$$

- heat transfer in Newtonian fluids

$$Nu = B(\text{géométrie}) Re^x Pr^y \left(\frac{\mu}{\mu_p}\right)^z$$

- heat transfer in Non-Newtonian fluids

$$Re_{a.eq} = \frac{\rho N d^2}{K \dot{\gamma}_{eq}^{n'-1}} = \frac{\rho N d^2}{K (K_s N)^{n'-1}} = \frac{\rho N^{2-n'} d^2}{K.K_s^{n'-1}}$$

The viscosity has been replaced by the equivalent viscosity (determined by the Metzner-Otto method).

Examples of correlations:

	D	D/d	p/d	ha/d	Fluide	
Nagata [1]	0,30	1,05 - 1,25	1	1	N, RF	$Nu = 1,39\ Re_e^{1/3}\ Pr_e^{1/3}\ Vis_e^{0,2}\ (e/D)^{-1,3}$ $1 < Re_e < 1\,000$ (76)
Mitsuishi [35] (2)	0,40	1,053	1	1,5	N, RF	$Nu = 0,78\ Re_4^{1/3}\ Pr_4^{1/3}\ Vis_4^{0,18}$ $1,5 < Re_4 < 10$ (77) $Nu = 0,53\ Re_4^{1/2}\ Pr_4^{1/3}\ Vis_4^{0,14}$ $10 < Re_4 < 180$ (78) $Nu = 0,23\ Re_4^{2/3}\ Pr_4^{1/3}\ Vis_4^{0,14}$ $180 < Re_4 < 4\,000$ (79)
Shamlou [6]					N, RF	$Nu = 0,17\ Re_e^{0,16}\ Pr_e^{1/3}\ Vis_e^{0,19}\ n_R^{0,22}\ (e/d)^{-0,45}\ (p/D)^{-0,24}$ $Re_{cr} < Re_e < 1$ (80) $Nu = 0,45\ Re_e^{0,6}\ Pr_e^{1/3}\ Vis_e^{0,19}$ $10 < Re_e < 1\,000$ et $Re_e > Re_{cr}$ (81)

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